The ETAS aftershock model

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1 | The ETAS model

The Epidemic Type Aftershock Sequence (ETAS) model is used to extend Poisson process models of earthquakes of magnitude m_0 and larger. The ETAS model allows for dependence between events and self excitation; earthquakes can cause further earthquakes.

For an ETAS point process the conditional earthquake intensity at location (x, y, t) is a function of spatial/temporal covariates X, the history of the process $\mathcal{H}_t$, and θ a vector of parameters to be estimated:

$$\underbrace{\lambda(x,y,t|X,\mathcal{H}_t,\theta)}_{\text{All earthquakes}} = \overbrace{\mu(x,y,t|X,\theta)}^{\text{Poisson mainshocks}} + \underbrace{\sum_{i:t_i<t} \kappa(m_i|\theta) g(t-t_i|\theta) h(x-x_i, y-y_i|\theta)}_{\text{ETAS aftershocks}}\,.$$

- $\mu(\cdot)$ is an intensity function, possibly varying based on a covariate.
- $\kappa(m)$ controls the expected number of aftershocks produced by an earthquake of magnitude m.
- $g(t)$ and $h(x, y)$ are probability density functions and control displacement of aftershocks in time and space.

2 | Conventional choices of κ, g and h

The mean count function, κ, is usually a simple thresholded montonic function.

$$\kappa(m|K,a) = K\exp\{a(m-m_0)\} \quad \text{for } m > m_0, \quad \text{where } K, a \geq 0.$$

The displacement distributions $g(t)$ and $h(x, y)$ are typically independent power-law distributions, with $h(x, y)$ isotropic so that $x^2 + y^2$ is power-law distributed. Power-law distributions are heavy tailed and have densities of the form:

$$g(t) = \begin{cases} (p-1)\,c^{p-1}\,(t+c)^{-p} & \text{for } t > 0, \\ 0 & \text{otherwise.} \end{cases} \quad \text{where } c > 0,\ p > 1,$$

The magnitude distribution f is conventionally independent of the rest of the process and represented by a shifted exponential distribution.

$$f(m|\beta) = \beta\exp\{\beta(m-m_0)\} \quad \text{for } m \geq m_0, \quad \text{where } \beta > 0.$$

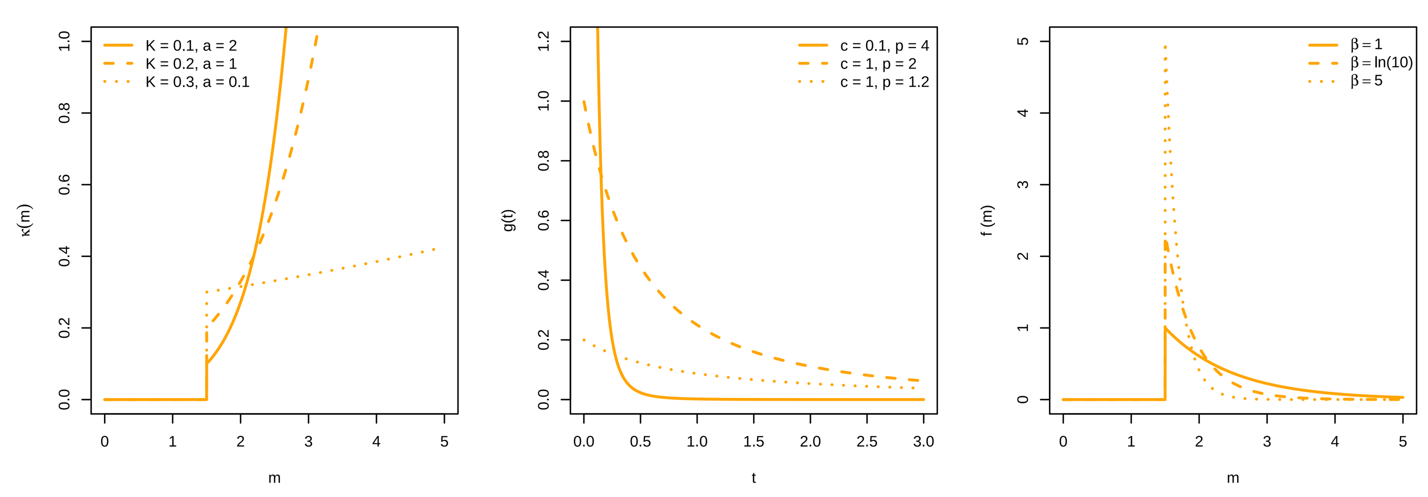

Figure 1: Example mean count, displacement and magnitude functions.

3 | Issues with conventional choices

Displacement & mean count functions:

- Highly correlated parameters (Fig 2)
- Interpretation of parameters is difficult or dependent on threshold chosen
- Distributions are constrained to be heavy tailed.

Magnitude model:

- Continuous model used for discrete observations, inducing bias
- Can have arbitrarily large magnitudes
- Ignores uncertainty in shape of tail.

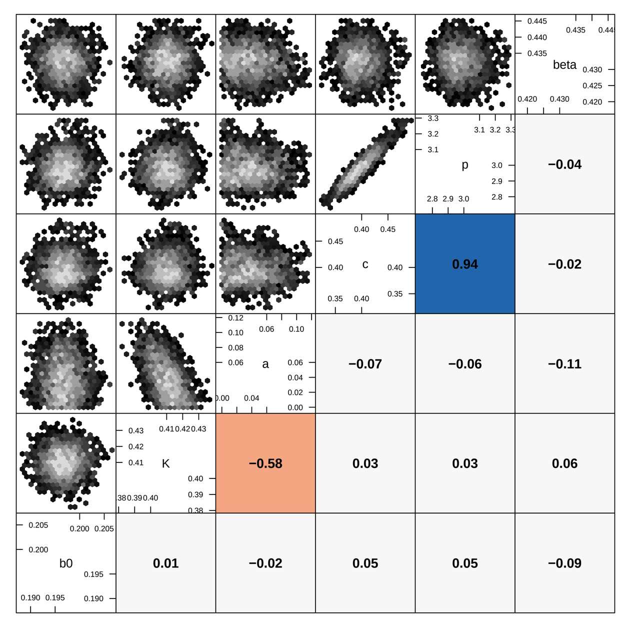

Figure 2: Posterior samples & correlations using conventional parameterisation.

Aftershock reparameterisation

- Centre the expected count function about $\bar{m}$.

$$K\exp\{a(m-m_0)\} = K\exp\{a(\bar{m}-m_0)\}\exp\{a(m=\bar{m})\} = C\exp\{a(m-\bar{m})\}$$

- New parameters C and a are independent.

- Fit using generalised Pareto distributions and then convert to power-law
- Power-law is equivalent to GPD with constrained parameters:

$$\nu = \frac{c}{p-1}\left(1+\frac{1}{p-1}\right), \quad \xi = \frac{1}{p-1}, \quad \text{with } \xi > 0.$$

- ν and ξ are independent for $\xi > -\frac{1}{2}$, therefore for all power-laws.

Magnitude Model

- Account for rounding by modelling observations using multinomial distribution.
- Incorporate uncertainty in tail behaviour by modelling unobserved true magnitudes as GPD.

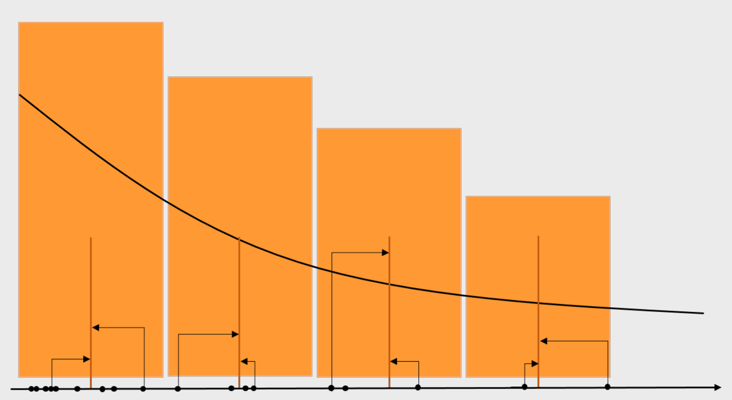

Figure 3: Schematic diagram of latent GPD multinomial model.

4 | Centred GPD model

Under the new model parameter dependence is reduced (Fig 4). This makes MCMC sampling schemes more efficient and when samples are transformed back to ETAS parameters, all comparable pairs have higher effective sample size (Tab 1).

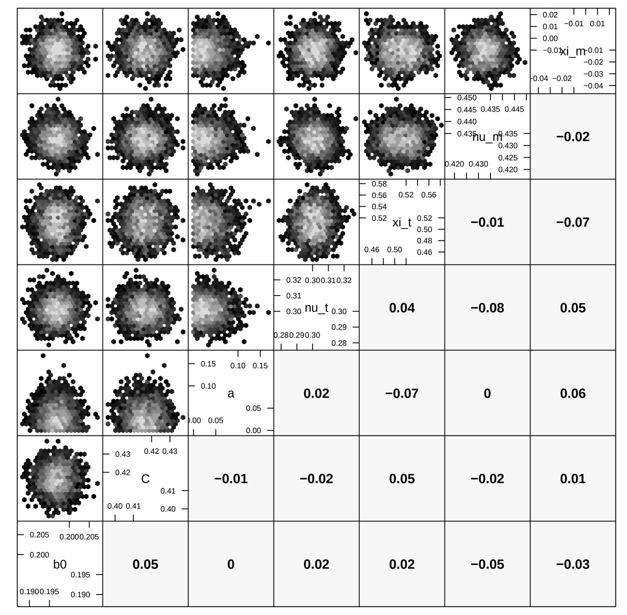

Figure 4: Posterior samples & correlations using proposed parameterisation.

Parameter	b_0	K	a	c	p	σ_m	ξ_m
ESS ETAS	3184	1727	486	522	521	5655	0
ESS cGPD	7544	3433	3117	729	622	2198	1257

Table 1: Effective samples sizes for conventional and proposed parameterisations.

Additional benefits of using centred GPD parameterisation:

1. Independent parameters simplify model interpretation,
2. Can extend parameter space to include light or short tailed densities,
3. Can accommodate a variable m_c easily,
4. Magnitude model accounts for decay uncertainty and respects physicality.

5 | Take-aways & Further work

Take aways:

A simple reparameterisation of the conventional ETAS model can make inference easier and results in an easier to interpret model.

Modelling magnitudes as exponential induces bias and ignores uncertainty in the tail behaviour. Instead, we can use a latent generalised Pareto model.

Further work:

- Investigate weaker clustering and variable background intensities
- Choice of covariate based background intenisty
- Include estimation of earthquake ancestry (stochastic declustering)
- Combine and apply to observed catalogues

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